THEORETICAL INVESTIGATION OF THE ELECTRODE POTENTIAL LAYER TAKING SURFACE IONIZATION AND THE SCHOTTKY EFFECT INTO ACCOUNT

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The results of an investigation of the characteristics of the electrode layer in a lithium plasma as a function of the parameters of the plasma surrounding the electrode are presented.

The layer close to the electrode is schematically represented as consisting of two regions: the free-fall region, in which the electric potential undergoes considerable changes, and a region in which molecular collisions occur.

In this paper we consider the free-fall layer taking surface ionization and the Schottky effect into account. Other phenomena can also play a decisive role. Surface ionization occurs when an alkali-metal plasma comes in contact with incandescent materials, and the Schottky effect occurs at increased pressure.

These problems have already been touched upon in some published papers. An expression was obtained in [1] for calculating the surface ionization at the boundary of the layer using an unjustified representation of the density distribution of the charged particles in the plasma close to the boundary, assuming $n_e > n_i$ when there is no electric current at the boundary. The surface ionization on the walls in the plasma was considered in a thorough manner in [2], but ignoring the Schottky effect. In [3, 4], devoted to a broader consideration of the collisionless layer, the relations derived are not suitable for taking the Schottky effect into account. In addition, no indication of surface ionization was given, and some inaccuracies were permitted in writing down the boundary conditions. A numerical example was given for only the simplest case. In [4], which removed the inaccuracies in [3] and complemented it, no results of calculations were given. In [5], devoted to an investigation of surface ionization in the plasma taking the Schottky effect into account, extremely rough expressions were given for the flow of particles at the external boundary of the free-fall layer, the plasma was considered as being in ionization equilibrium, and the calculations were made for the special case of a cesium plasma with a low degree of ionization.

Below we obtain a solution of the problem of the Langmuir layer taking into account such important factors that determine the state of the plasma–electrode system as the Schottky effect, surface ionization, and the lack of symmetry in the particle distribution function at the external boundary of the layer. Numerical investigations of the Langmuir layer carried out on a computer over a wide range of variation of the current density and degree of ionization of the plasma enable us to demonstrate the overall nature of the behavior of the current–voltage characteristics of the layer. We only consider the case of a monotonic change in the potential in the electrode layer. We carried out our investigations on a lithium plasma, lithium being an element which belongs to the technically important group of alkali metals, and which is widely used for practical and research purposes [6-8].

Consider a plane collisionless layer between a plasma of pressure p_s and degree of ionization α_s and a thermionic electrode for the case when the potential jump in the layer $\Delta\varphi=\varphi_s-\varphi_w > 0$, assuming that the electron gas at the external boundary of the layer is simulated by a distribution with a mean temperature T_{es}, which differs from the temperature of the heavy particles which are in equilibrium with the electrode.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 28, pp. 482-489, March, 1975. Original article received June 24, 1974.

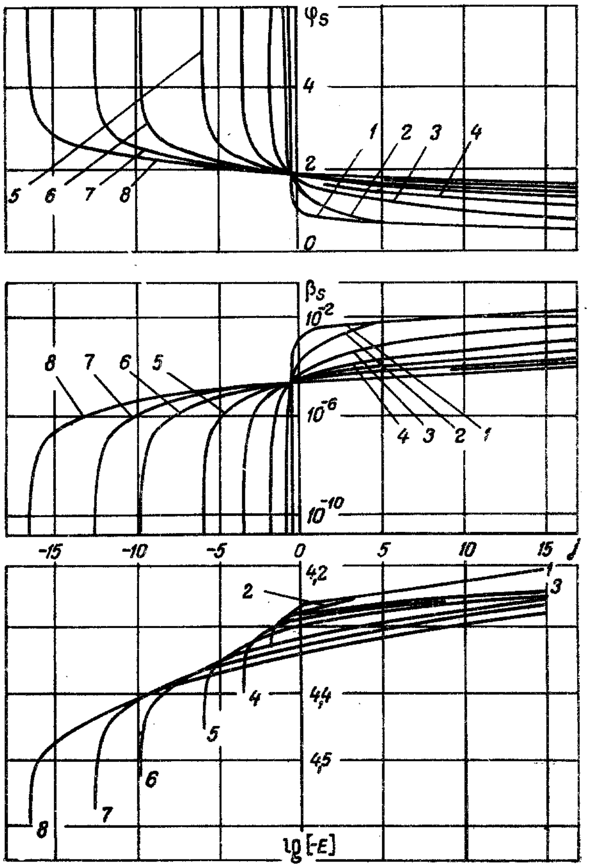

Fig. 1. Curves of the electrode fall, the effective surface ionization coefficient on the external boundary of the layer, and the electric field at the surface as a function of the current to the electrode ($p_s = 10^3$ μbar, $T_w = 2.5 \times 10^3$ K, $T_{es} = 5 \times 10^3$ K): 1 - $\alpha_s = \beta_s/2$, 2 - 5×10^{-3}, 2×10^{-2}, 4 - 5×10^{-2}, 5 - 1×10^{-1}, 6 - 2×10^{-1}, 7 - 3×10^{-1}, 8 - $(1 + \beta_s)/2$. φ_s, V, j, A/cm^2; E, V/cm.

The overall current density at the electrode is made up of the sum of the electron and ion currents

$$j = j_e + j_i. \tag{1}$$

The electron current density is equal to the sum, with the appropriate sign, of the thermionic current density and the electron current density which crosses the potential barrier

$$j_e = -j_{em} + e\nu_{es}^{-} \exp\left(-\frac{e\Delta\varphi}{kT_{es}}\right), \tag{2}$$

where $\bar{\nu}_{es}$ is the electron flux through the external boundary of the layer s towards the electrode, j_{em} is the thermionic current density, given by the Richardson–Schottky equation

$$j_{em} = AT_w^2 \exp\left(-\frac{e(\varphi_e^* - \sqrt{eE_k})}{kT_w}\right), \tag{3}$$

T_w is the electrode temperature, $e\varphi_e^*$ is the thermionic work function, and E_k is the electric field strength at the surface of the electrode.

The ion current density is

$$j_i = e(\nu_{is}^{+} - \nu_{is}^{-}), \tag{4}$$

where $\nu_{is}^{\pm}$ are the ion fluxes through the boundary s in the direction of the external and internal normal respectively.

The particle fluxes $\nu_{is}^{\pm}$ are related to the particle densities and the thermal velocities as follows:

$$\nu_{js}^{\pm} = \frac{1}{4}\zeta_{js}^{\pm} n_{es} v_{js}, \tag{5}$$

where n_{es} is the electron density at the boundary of the layer, v_{js} is the thermal velocity, and the subscript j takes the value e and i. At the boundary s we assume $(n_{is} - n_{es})/n_{is} \ll 1$.

The appearance of the factor ζ_{js} in Eq. (5), which takes account of the lack of symmetry of the particle velocity distribution function at the external boundary of the layer, is due to the fact that in the majority of gas-discharge modes the particle distribution is not Maxwellian. A detailed derivation of the equations for determining the coefficients ζ_{js} taking into account surface ionization and elastic reflection of the particles from the electrode surface is carried out in [9]. Here we will merely quote the final results:

$$\zeta_{es}^{-} = \frac{2 - j_{em}\Big/\frac{1}{4} e n_{es} v_{es}}{2 - (1-\gamma_e)\exp\left(-\frac{e\Delta\varphi}{kT_{es}}\right)}, \tag{6}$$

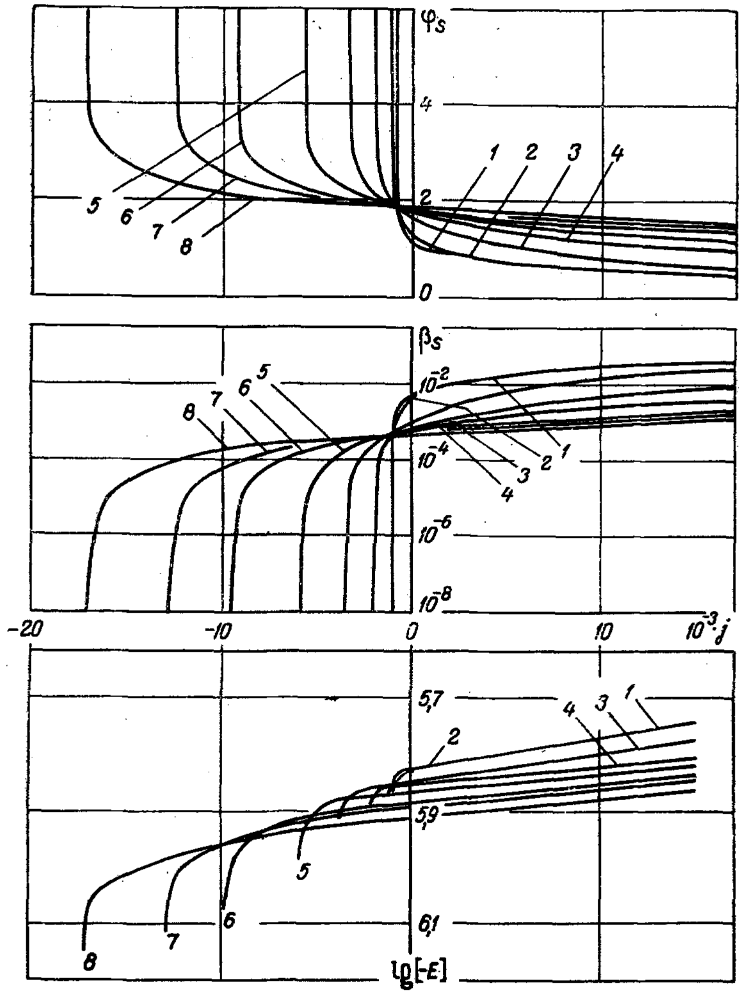

Fig. 2. Electrode fall in potential, effective surface ionization coefficient at the external boundary of the layer, and the electric field strength at the surface as a function of the current to the electrode ($p_s = 10^6$ μbar, $T_w = 3.5 \times 10^3$ K, and $T_{es} = 5 \times 10^3$ K): 1 - $\alpha_s = \beta_s/2$, 2 - 5×10^{-3}, 3 - 2×10^{-2}, 4 - 5×10^{-2}, 5 - 10^{-1}, 6 - 2×10^{-1}, 7 - 3×10^{-1}, 8 - $(1 + \beta_s)/2 \cdot \varphi_s$, V; E, V/cm, j, A/cm^2

$$\zeta_{is}^{-} = 2 - \frac{\beta_s}{\alpha_s}, \quad \zeta_{is}^{+} = \frac{\beta_s}{\alpha_s}, \tag{7}$$

where β_s is the effective surface ionization coefficient, which relates the flux of the ion component leaving through the boundary s, with the total flux of heavy particles at the electrode [9]

$$\beta_s = \{2\alpha_s\,[\gamma_i + \beta_s'(\gamma_a - \gamma_i)] + (1 - \gamma_a)\,\beta_s'\}\,[1 + \gamma_i + \beta_s'(\gamma_a - \gamma_i)]^{-1}, \tag{8}$$

γ_a, γ_i, γ_e are the elastic reflection coefficients of the atoms, ions, and electrons respectively, and β_s' is given by the expression

$$\beta_s' = \beta_w \exp\left(-\frac{e\Delta\varphi}{kT_w}\right)\left\{1 - \beta_w\left(1 - \exp\left(-\frac{e\Delta\varphi}{kT_w}\right)\right)\right\}^{-1}. \tag{9}$$

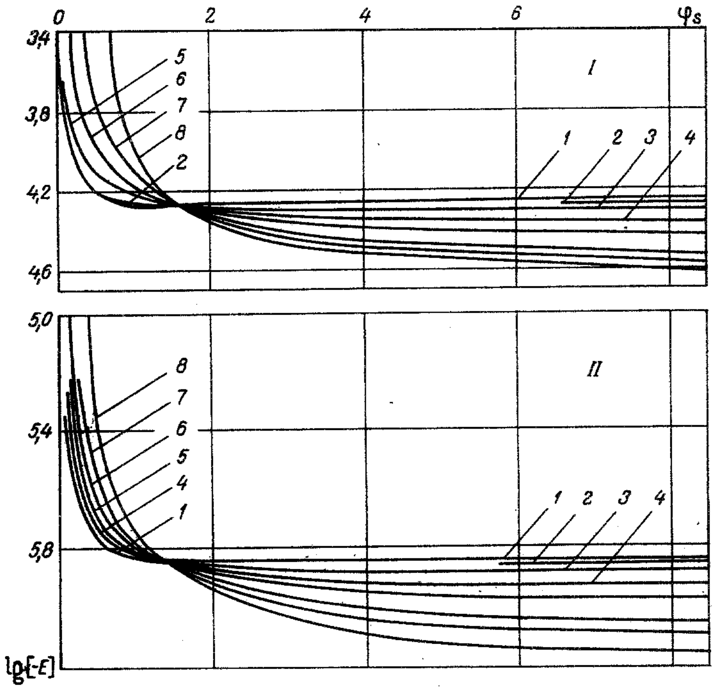

Fig. 3. Curves of the electric field strength at the surface of the electrode as a function of the electrode fall in potential (I – $p_s = 10^3$ μbar, $T_w = 2.5 \times 10^3$ K, $T_{es} = 5 \times 10^3$ K; II – $p_s = 10^6$ μbar, $T_w = 3.5 \times 10^3$ K, $T_{es} = 5 \times 10^3$ K): 1 – $\alpha_s = \beta_s/2$; 2 – 5×10^{-3} 3 – 2×10^{-2}; 4 – 5×10^{-2}, 5 – 1×10^{-1}, 6 – 2×10^{-1}, 7 – 3×10^{-1}, 8 – $(1 + \beta_2)/2$·E, V/cm, φ_s, V

The surface ionization coefficient β_w is found from the Saha–Langmuir formula.

Substituting Eqs. (2), (4), and (5) into Eq. (1) we obtain the following expression for the total current density at the electrode:

$$j = \frac{1}{4} \zeta_{es} e n_{es} v_{es} \exp\left(-\frac{e\Delta\varphi}{kT_{es}}\right) - j_{em} - \frac{1}{2} e n_{es}\left(1 - \frac{\beta_s}{\alpha_s}\right) v_{is}. \tag{10}$$

The last equation enables us to follow the dependence of the electrode potential fall in the layer $\Delta\varphi$ on the total current density for given plasma parameters and electric field strengths E_k, which in turn is determined by the state of the plasma-electrode system.

To determine E_k we must use the Poisson integral equation, which when $E_s^2 \ll E_k^2$ can be written in the form

$$E_k^2 = 8\pi \int_{\varphi_w}^{\varphi_s} \rho d\varphi. \tag{11}$$

The problem of determining the total space-charge density at each point of the range $[\varphi_w, \varphi_s]$ is considered in detail in [9].

Equation (6), (8), (10), and (11) were solved on a computer for quasi-stationary conditions in a Li plasma for the following conditions: $P_s = 10^3$ μbar, $T_w = 2.5 \times 10^3$ K, and $p_s = 10^6$ μbar, $T_w = 3.5 \times 10^3$ K. In both cases $T_{es} = 5 \times 10^3$ K. We used tungsten as the electrode material with the following characteristics: the constant in the Richardson–Dushman equation A = 100, thermionic work function $e\varphi_e^* = 4.58$ eV, and surface ionization work function $e\varphi_i = 5.14$ eV [10–11]. The electron elastic reflection coefficient from the electrode surface is 0.1 [12]. We took the potential of the electrode surface to be zero. Then

$\Delta\varphi = \varphi_s - \varphi_w = \varphi_s$. We calculated the dependence of φ_s, β_s, and E_k on j both for the limiting possible values of α_s, and for certain constant values of the degree of ionization: 5×10^{-3}, 2×10^{-2}, 5×10^{-2}, 10^{-1}, 2×10^{-1}, and 3×10^{-1} in the range of currents including both cathode conditions ($j < 0$) and anode conditions ($j > 0$). The limiting possible values of α_s correspond to the condition when the ion flux ($\alpha_s = \beta_s/2$) or the flux of atoms [$\alpha_s = (1 + \beta_s)/2$] on the electron surface is zero. For values of the degree of ionization at the boundary s which fall outside the range [$\beta_s/2$, $(1 + \beta_s)/2$], the discharge cannot exist (for the given plasma parameters and chosen material and electrode temperature).

Figures 1 and 3, I show the results obtained for a pressure $p_s = 10^3$ μbar, and Figs. 2 and 3, II show the results obtained for $p_s = 10^6$ μbar. Analysis of the results obtained enables us to draw the following conclusions.

The electrode fall in potential depends to a considerable extent on the degree of ionization of the surrounding plasma. For the curves of φ_s, β_s as a function of j the curves α_s = const and also $\alpha_s = \beta_s/2$, $\alpha_s = (1 + \beta_s)/2$ intersect in a narrow region of values of j, adjacent to the value $j = j_{em}$ on the right side. For example, when $p_s = 10^3$ μbar the spread of the points of intersection is less than 0.7% of j_{em}. The point of intersection of the curves divides the current into two regions. All the curves $\varphi_s(j)$ and $\beta_s(j)$ for α_s = const lie between the limiting curves $\alpha_s = \beta_s/2$ and $\alpha_s = (1 + \beta_s)/2$.

When $j < j_{em}$, which relates to the case when the electrode operates as a cathode, an increase in the degree of ionization α_s causes a reduction in φ_s and an increase in β_s for a given j. The curves of α_s = const for the current density $j = j_{em} + e/4\ \zeta_{is} n_{es} v_{is}$ reach asymptotes. To the right of the point of intersection there is both a cathode region with currents $0 > j > j_{em}$, and an anode region. Here, as the degree of ionization increases a different behavior is observed consisting of an increase in φ_s and a decrease in β_s as α_s increases. In this current region the curves of α_s = const are confined by the limiting curve $\alpha_s = \beta_s/2$. The anode region is characterized by increased values of the effective surface ionization coefficient β_s. In the modes investigated β_s reaches several percent. In the range of currents considered at the anode $\Delta\psi > 0$.

An analysis of the values of the electric field strength E_k at the surface under different modes of operation shows that α_s has an effect on E_k. Under anode conditions and particularly under cathode conditions, an increase in α_s causes an increase in $|E_k|$. Under cathode conditions, there is a small range of the parameters j and α_s in which an increase in α_s causes a reduction in $|E_k|$.

In φ_s, log $(-E_k)$ coordinates the curves of α_s = const, and also the curves $\alpha_s = \beta_s/2$ $(1 + \beta_s)/2$ form two regions divided by the point of intersection. In the region of high values of φ_s an increase in the degree of ionization causes an increase in $|E_k|$, while in the region of small values of φ_s an increase in the degree of ionization causes a reduction in $|E_k|$.

Within the limits of the assumed variation in the current density, E_k varies over quite a narrow range. For $p_s = 10^3$ μbar this variation amounts to

$$\frac{21}{22} \lg(-E_{av}) < \lg(-E_k) < \frac{23}{22} \lg(-E_{av}),$$

while for $p_s = 10^6$ μbar it amounts to

$$\frac{13}{14} \lg(-E_{av}) < \lg(-E_k) < \frac{15}{14} \lg(-E_{av}).$$

A comparison of data obtained at different pressures shows that when the pressure is increased from 10^3 to 10^6 μbar the point of intersection of the curves in Fig, 3, I and II shifts from 1.6 to 1.4 V. The average field strength, according to Figs. 1 and 2, increases from log $(-E_k)$ = 4.4 to log $(-E_k)$ = 5.95. Whereas at a pressure of 10^3 μbar the correction to the thermionic emission current due to the Schottky effect is (27-46)%, at a pressure of 10^6 μbar it is (157-353)%. The Schottky effect has a considerable effect on the curves of φ_s, E_k, and β_s as a function of j for $p_s = 10^6$ μbar.

A comparison of Figs. 1 and 2 shows that we can obtain larger currents for approximately the same potential drops as the pressure is increased.

We checked the effect of surface ionization on the electric field at the surface by calculating E_k using the same equations for the electrode layer without surface ionization. For $\varphi_s = 2$ V, $p_s = 10^3$ μbar, and $T_w = 2.5 \times 10^3$ K for a variation of α_s from $(1 + \beta_s)/2$ to $\beta_s/2$ the field strength $|E_k|$ taking surface ionization into account increased by a factor of from 1.22 to 51, while for the same φ_s and $p_s = 10^6$ μbar, and $T_w = 3.5 \times 10^3$ K it increased by a factor of from 1.28 to 70. The current density in the first case increased by a factor of from 1.05 to 1.23, and in the second case it increased by a factor of from 1.025 to 2.46. A consideration of the space-charge distribution in the layer shows that surface ionization leads to greater fullness of the profile by the positive charge in the case of large values of α_s, and even to a change in the sign of the charge at the surface for small values of α_s.

The results presented above give a general representation of the effect of surface ionization and the Schottky effect on the characteristics of the Langmuir layer. They can be used both in practical calculations of the physical state of a plasma and to interpret the experimental data.

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